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Study of Fast Ion Confinement in Reversed Field Pinch

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This work is intended for assessing possible merits of the NBI heating scheme in the MST reversed field pinch. Good confinement of beam ions is obviously necessary for efficiency of such heating.

In RFPs, there are two significant features that make confinement of fast ions different than in a typical tokamak. One is the relatively weak and non-uniform magnetic field, which causes the ion Larmor radius to be comparable to the gradient length, and the other is the inherent stochastisity of field lines over a significant part of the discharge. The first problem may cause a significant fraction of beam particles to be ionized on regular trajectories intersecting the wall, while the second one may lead to stochastic diffusion in radius.

The fraction of ions lost on regular trajectories can be studied in the axisymmetric-field model on the basis of conservation of the generalized toroidal momentum. This approach can be applied for arbitrary ion energies in contrast to the drift approximation. The most efficient injection aiming is found to be in the direction of the plasma current, approximately tangentially to the magnetic axis.

At present conditions in the MST, the long-term life-time of fast ions is determined by charge-exchange losses, while in future experiments the stochastisity of the magnetic field may become important. The question here is whether the drift trajectories of fast ions follow the stochasticity of the field lines. The answer depends on the resonance conditions between the ion trajectories and the helical structure of the magnetic perturbations. In particular, trapped particles interact with the stochastic field in a quite different way. For a typical injection scheme in the MST-like field most captured beam particles are either trapped initially, or will become trapped in the course of radial diffusion (with magnetic moments conserved).

If just two integrals of motion are present, such as in an axisymmetric torus without μ conservation, where just the energy $\varepsilon = \frac{m}{2}\left(v_r^2 + v_\varphi^2 + v_\theta^2\right) + e\Phi$ and the generalized toroidal momentum

$$P_\varphi = 2\pi R\left(mv_\varphi + \frac{e}{c}A_\varphi\right) \tag{1}$$

are conserved, the trajectory may be chaotic even if the magnetic field is regular. Here v and e are the velocity and the charge of the particle, Φ is the electrostatic potential, R is the major radius, A_φ is the φ-component of the vector potential. To estimate the radial extent of the trajectory one can write the inequality

$$\frac{2}{m}\left(\varepsilon - e\Phi(\psi)\right) - v_\varphi^2\left(P_\varphi, \psi, R\right) \geq 0, \tag{2}$$

which simply states that $v_r^2 + v_\theta^2 \geq 0$ in any point of the trajectory. Here we assume that the magnetic field has flux surfaces so that the φ-component of the vector potential A_φ can be expressed via the poloidal flux-function ψ=$2\pi RA_\varphi$, while $\Phi = \Phi(\psi)$.

The inequality (2) describes just a necessary condition for confined motion to be possible, that is, it yields an upper (worse than it could be) estimate of the motion range. However, if the particle motion is indeed chaotic, the trajectory can fill all available phase volume and reach boundaries defined by (2). The motion will not be chaotic and will be contained in a smaller radial range if there are additional integrals of motion such as $\widehat{\mu}$.

The influence of the electrostatic potential is relatively weak, if the change of the potential $e\delta\Phi$ along the ion trajectory is of the order of the temperature of the plasma core $T_c \sim 1$ keV, which is small compared to the injection energy ε: $e\delta\Phi/\varepsilon \sim T_c/\varepsilon \ll 1$. In this approximation the inequality (2) can be rewritten as

$$\left| R_i \cos\gamma + \frac{\psi_i - \psi}{2\pi B_0 \rho} \right| \leq R\left(1 + \widehat{\Phi}_i - \widehat{\Phi}\right), \tag{3}$$

where index "i" denotes values at the ionization point, so that v_i is the velocity of the ion at the initial point, γ is the angle of the initial velocity to the magnetic axis, $\cos\gamma = v_{\varphi i}/v_i$, B_0 is the magnetic field on the magnetic axis, $\rho = c\sqrt{2\varepsilon_i m}/eB_0$ is the characteristic Larmor radius, $\varepsilon_i = mv_i^2/2$ is the injection energy, and $\widehat{\Phi} = e\Phi/2\varepsilon_i \ll 1$ is the normalized electrostatic potential. It follows that for realistic potentials of the order of 1 kV the effect of the radial electric field on confinement of fast ions is negligible.

Model of the equilibrium

For estimates we shall assume that the toroidal discharge is contained in an axisymmetric shell with a circular cross-section with the inverse aspect ratio $\widehat{\epsilon} = a/R_0 = 0.3$, where R_0 is the radius of the toroidal axis of the shell, and a is its minor radius. Azimuthal cross-sections of the flux surfaces are assumed to be shifted circles, so that

$$R = R_0 + \Delta(r) + r\cos\theta, \qquad z = r\sin\theta, \tag{4}$$

while $\psi = \psi(r)$, $\Delta(r) = \Delta_0\left(1 - (r/a)^2\right)$, where $0 \leq \Delta_0 \leq 0.15$ is a variable parameter. Dependence of the magnetic field on the minor radius is chosen to follow the standard Bessel-function model[1], so that

$$\psi(r) = \frac{2\pi R_0 B_0}{\mu}(1 - J_0(\mu r)), \tag{5}$$

where μ is a variable parameter, $2 \leq \mu a \leq 3$. Results are found to be only weakly dependent on Δ_0 and μ.

Confinement on regular trajectories

The threshold equation (for particles just touching the wall) looks like

$$\left| \widehat{R}_i \cos\gamma - \frac{1}{\mu\rho}\left(J_0(\mu r_i) - J_0(\mu a)\right) \right| = 1 + \epsilon, \tag{6}$$

where $\epsilon = r_i/R_0$, $\widehat{R}_i = R_i/R_0$ and satisfies $\widehat{R}_i = 1 + \epsilon\left(\Delta(r_i) + r_i\cos\theta_i\right)/a$. Figures below correspond to the field model with $\Delta_0 = 0.1$, $\mu a = 2.6$.

Note the up-down symmetry in the poloidal angle, θ_i. This symmetry is somewhat contrary to intuitive expectations, since the pitch-angles for θ_i and $-\theta_i$ may be quite different. Using additional invariants such as $\widehat{\mu}$ or P_θ will indeed cause breakdown of the symmetry.

We solve the above equations to 1) find boundaries of confinement of fast ions with different initial positions and velocities; 2) find the effective beam absorption length L_{eff} for different injector positioning and aiming.

Figure 1 shows that beam particles injected along the magnetic axis in the direction of the plasma current ($\gamma = 180^o$) are well confined up to $\rho/a \sim 1$, while if their initial velocity is in the opposite direction, confinement occurs only for particles with $\rho/a > 6$, and is generally worse. Particle simulation for $a/\rho = 8$ illustrates the fact that the conservation of the magnetic moment improves confinement. Note also, that in the *drift* approximation confinement is generally better, since the radial drift velocity is exactly zero on the field-reversal surface.

Figure 2 shows L_{eff}/R_0 for a thin-beam injection within the equatorial plane, and the aiming-optimization figure for the actual injector-port position, $a/\rho = 6$. Both of these show that injection approximately tangent to the magnetic axis in the direction of the plasma current is favorable for confinement.

Confinement in perturbed fields

The above approach is counter-intuitive in that it assumes conservation of P_φ and neglects conservation of $\widehat{\mu}$, while the usual case (for tokamaks) is exactly the opposite. In general, the axial symmetry of the magnetic field (and thus $P_\varphi = \mathrm{const}$) is broken by helical perturbations caused by the plasma turbulence. These perturbations being small, the resonance conditions between the helical structure of the perturbations and the helical structure of the drift trajectories are necessary for a significant change in P_φ to occur. Furthermore, overlapping of such resonances for different modes is necessary for the phase-space diffusion (Chirikov criterion). It is well known that for the magnetic field lines in RFPs this criterion is satisfied in a broad region of the minor radius, and thus the radial diffusion of field lines and *particles*, if they are moving exactly along them, occurs. However, there are differences in behavior of field lines and particle trajectories, especially for large Larmor radii.

One such effect, namely, the averaging of the perturbation over the particle trajectory around the field line, is insignificant. Indeed, simulations and measurements show the radial and poloidal wavelength to exceed the possible Larmor radius. The other one, namely, deviation of the drift trajectory from the field line, may be important.

The drift trajectory has generally a different rotational transform than the field lines and may cease to be in resonance with the spectrum of helical perturbations. Also, if the particle is trapped, its average motion is just in the toroidal direction and ceases to resonate with helical fields altogether, if the toroidal drift frequency of the banana is different from that of the perturbations. For fast ions, just those particles, which have mirror points (trapping angle) close to 90^o will be lost.

For a typical injection aiming, around 1/3 of particles is trapped initially, while further 1/3 will become trapped in the course of radial diffusion if it occurs with conservation of $\widehat{\mu}$. They can be expected to avoid radial diffusion along the braided magnetic field lines of the RFP.

[1] J.B.Taylor, Reviews of Modern Physics, **58**, p. 741 (1986)

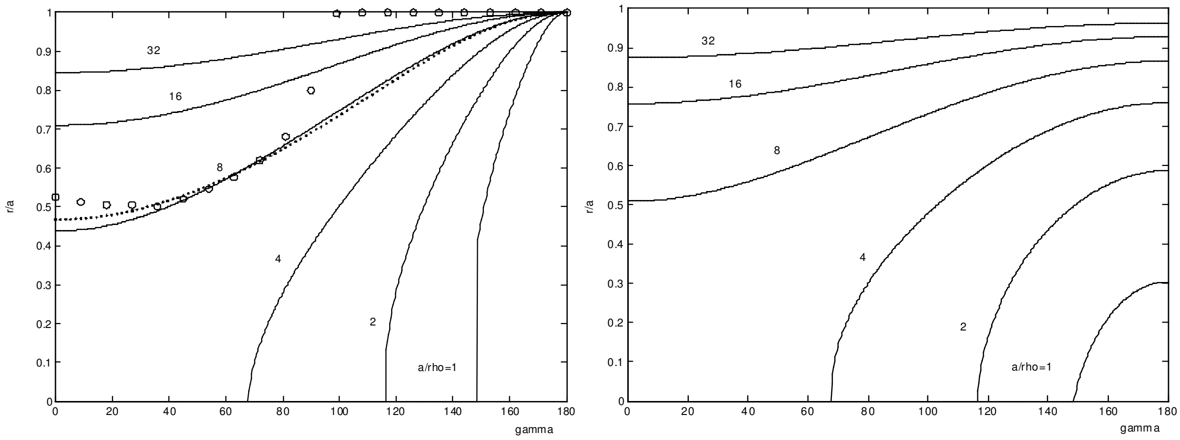

Fig.1. Confinement boundaries versus the injection angle to the magnetic axis for different Larmor radii. Line of dots: for fields from the NIMROD code. Circles: particle simulation, 100 turns. Figures are for different positions of the ionization point, $\theta_i = 0$, $\theta_i = \pi$.

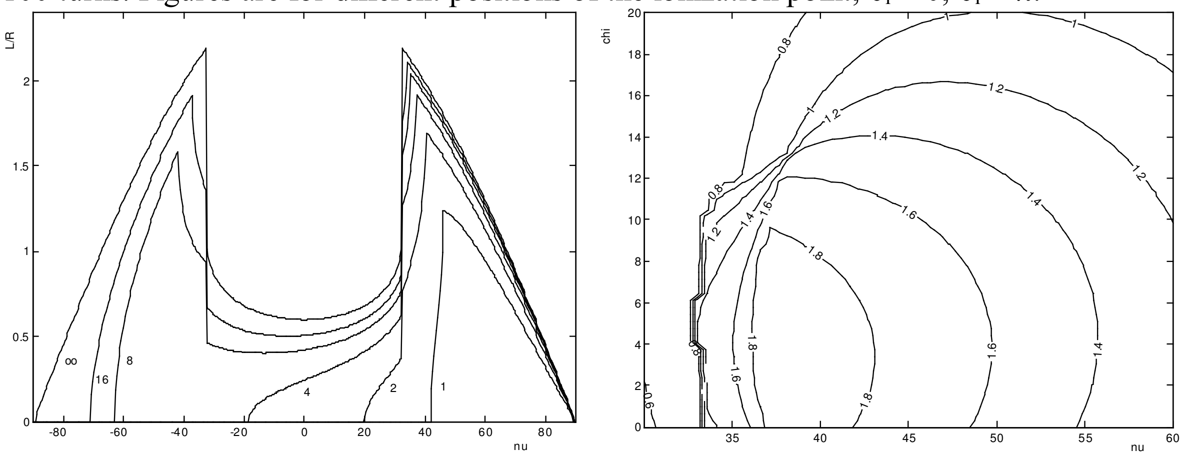

Fig. 2. The effective trapping length L_{eff} in the equatorial plane vs. injection angle and for a port position at 19^0 above the equatorial plane vs. inclination (chi) and azimuthal (nu) angles.

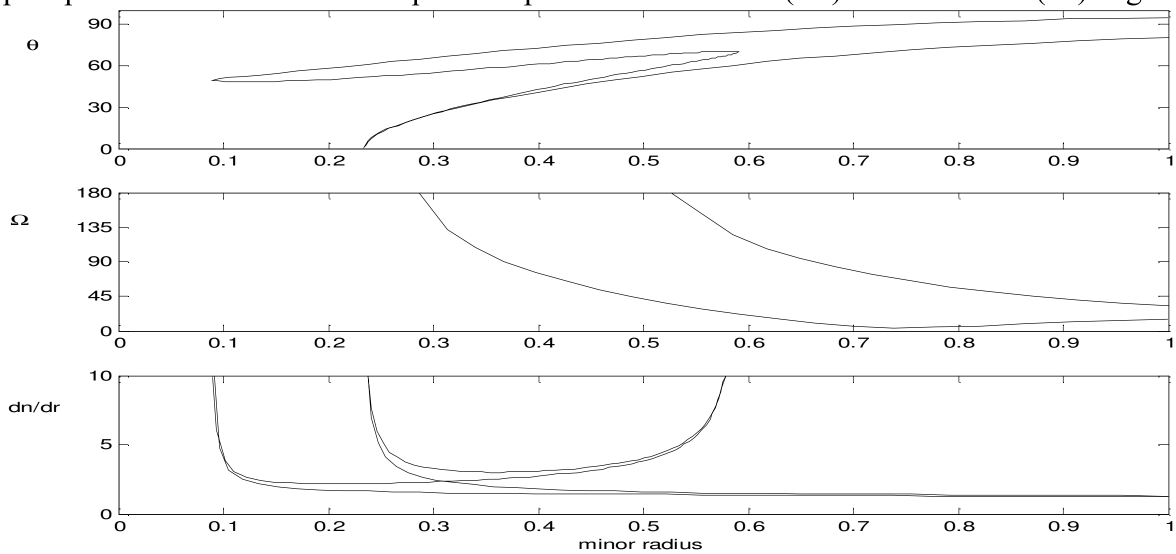

Fig. 3. The pitch angle θ, the trapping depth Ω, and the trapping density *dn/dr* vs. minor radius for the optimized injector aiming of Fig.2.